UDC 624.04+539.3

PROPAGATION OF ELASTIC TRANSIENT WAVES IN 1D PERIODICALLY HETEROGENEOUS STRUCTURES

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Nowadays, periodically heterogeneous structures and materials are widely used in Civil Engineering. As practical examples, one can note multi-span beams, perforated membranes and plates, stringer plates and shells, and a large variety of composite materials.

Propagation of elastic waves in heterogeneous structures is accompanied by a number of remarkable dynamic effects, such as phononic band gaps, negative refraction, dynamic anisotropy and waves focusing, acoustically invisible cloaks, waves localization in structures with defects, and splash effects. For extended survey of the progress in the field, we refer to Hussein et al. [1].

In this paper we study elastic waves propagating in 1D periodically heterogeneous structures (i. e., in a discrete monatomic lattice and in a continuous layered structure) subjected to an external pulse load. The important feature of the pulse load problem is that during the transient wave propagation the local stresses induced on the microlevel can exceed sufficiently the magnitude of the initial excitation. This effect is caused by a spatial redistribution of energy due to the heterogeneity of the structure and it can never be observed in homogeneous solids.

Let us consider a semi-infinite ($x \geq 0$) lattice consisting of identical particles of the mass m connected by massless springs of rigidity c (Fig. 1). The pulse load $P\delta(t)$ is applied to the lattice at the edge $x = 0$. Here P is the amplitude of the force and $\delta(t)$ is the Dirac delta function.

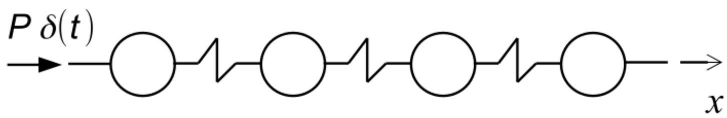

Fig. 1. Monatomic lattice under consideration

Colquitt et al. [3] proposed a higher-order triple-dispersive dynamic equation describing the macroscopic behavior of the lattice in a wide frequency range. We employ this continuous model to derive the analytical solution of the pulse load problem using the method of Laplace transform [4]. The numerical solution of the original discrete problem is developed by the Runge-Kutta fourth-order method. The obtained results for the displacements u are presented at Fig. 2 (the calculations are performed for $P/(cl)=1$, $t\sqrt{cm}=1$, where l is the distance between the particles). The analytical and the numerical solutions show a good agreement.

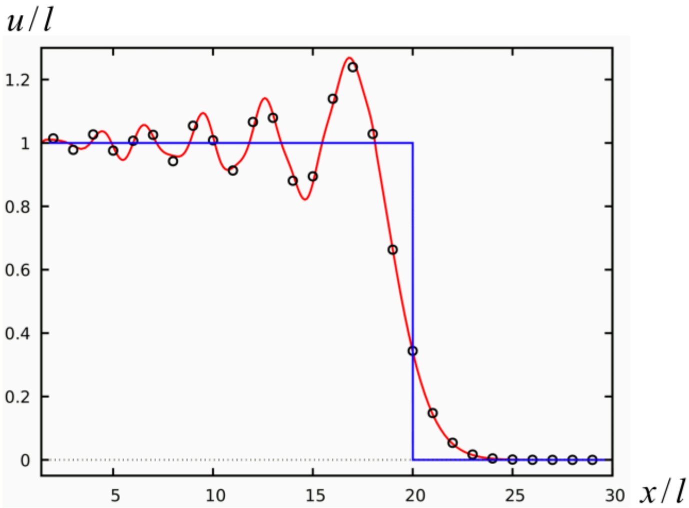

Fig. 2. Transient waves in the lattice excited by the pulse load. Red – analytical solution; blue – non-dispersive solution for the homogeneous solid; dots – data of the numerical simulation

Propagation of elastic waves in a continuous periodically heterogeneous structure is simulated numerically using ANSYS. The structure consists of alternating layers of two different materials. The model under consideration includes 100 unit cells of the dimensions 1×1×1 m; the thickness of each layer is 0.5 m and the entire length of the structure is 100 meters (Fig. 3).

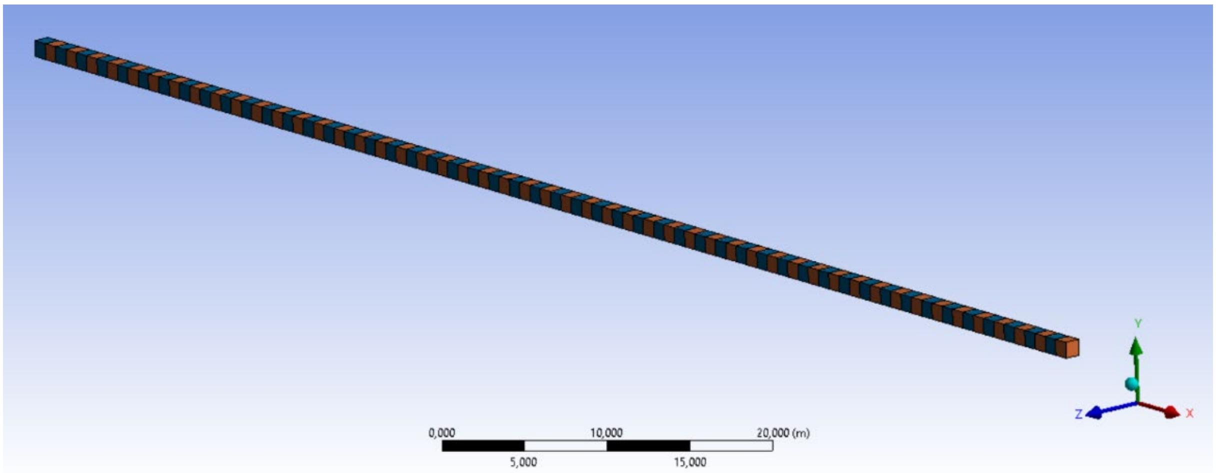

Fig. 3. Basic geometry of the continuous layered structure

Transient waves propagating in the direction x are studied. The macroscopic boundary conditions are as follows: the fixed support at one edge of the structure, the pulse load of 1000 N applied in the x direction at the opposite edge, and the smooth support along the longitudinal faces. As an illustrative example, let us consider a concrete-rubber structure, which is a promising design for new types of vibration and seismic isolation systems [5; 6]. The properties of the layers are given in Table 1. Fig. 4 displays the time-depended solution for the normal strain in the cross section at the distance of 40 meters from the loaded edge.

Table 1

Layer	Material	Density, kg/m^3	Young's Modulus, Pa	Poisson`s Ratio
1	Concrete	2.3×10^3	3.1×10^{10}	0.33
2	Rubber	1.3×10^3	5.8×10^5	0.46

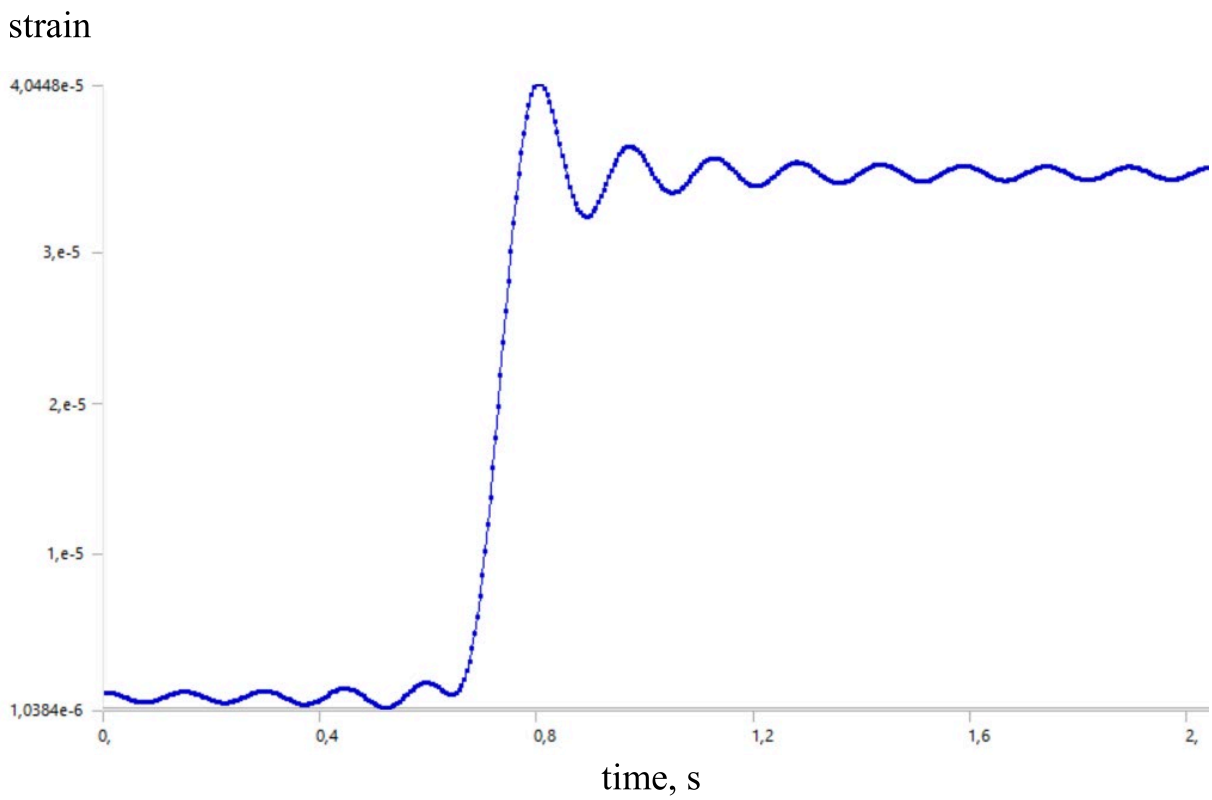

Fig. 4. Normal strain in the layered structure induced by the pulse load

Analysis of the results presented at Figs. 2 and 4 shows that in the heterogeneous structures local splashes of the displacement and strain fields can exceed the average magnitudes (which correspond to the homogeneous case) from 17 up to 24 %. This effect is crucial for the dynamic failure and is of great practical importance for the design of new heterogeneous structures and materials. The developed approaches can be also generalized to multi-dimensional problems.

Acknowledgement. This work is supported by the MSCA-RISE-2020 project "Effective Factorisation techniques for matrix-functions: Developing theory, numerical methods and impactful applications", grant agreement ID: 101008140.

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